

Final Exam
Chapters 1-6

Short Answer

Answer the following questions. *You must show your work to receive full credit.* Be sure to make reasonable simplifications. Indicate your final answer with a box.

1. (5 points) Let f be a differentiable function.

- (a) Give an interpretation of $f'(x)$.
- (b) How do we approximate $f'(x)$?

2. (5 points) Let f be an integrable function on $[a, b]$.

- (a) Give an interpretation of $\int_a^b f(x)dx$.
- (b) How do we approximate $\int_a^b f(x)dx$.

3. State or find the desired limits.

(a) (3 points) $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

(b) (3 points) $\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$

(c) (4 points) $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x$

4. (5 points) Find $\frac{dy}{dx}$ given that $y = \frac{\tan x}{\ln x} + 10$.

5. (5 points) Verify that the derivative of $g(x) = x \ln x$ is given by $g'(x) = \ln x + 1$. (Show all steps for credit.)

6. (3 points) Find $\int \ln x + 1 dx$. (Refer to previous problem.)

7. (10 points) Find all asymptotes of the function $f(x) = \frac{2x^2-2}{5x^2-40x+35}$.

8. (5 points) For what values of a is the function

$$m(x) = \begin{cases} a^2x - 2a & x \geq 2 \\ 12x & x < 2 \end{cases}$$

continuous at every x ?

9. (5 points) Find the tangent line to the curve $y = e^{1/x}$ at the point $x = 1$.

10. (5 points) Find $\frac{dr}{d\theta}$ given that $\theta^{1/2} + r^{1/2} = 1$.

11. (10 points) Find the global maximum and minimum for the function $f(x) = 4 - x^3$ on the closed interval $-2 \leq x \leq 1$.

12. (10 points) Determine where the function $f(x) = e^{2/x}$ is increasing/decreasing and where it is concave up/concave down.

13. A cargo ship is moving through outer space and its velocity function is given by $v(t) = \frac{2t}{t^2+1}$ light years per day.

- (a) (4 points) Find the ship's acceleration as a function of t . (Make sure to give units.)
- (b) (4 points) Find the ship's distance from Earth as a function of t assuming that it is initially 100 light years from Earth. (Make sure to give units.)
- (c) (4 points) Find the ship's average velocity in the first 10 days of its travels. (Make sure to give units.)

14. (5 points) Find the area between the curves $f(x) = 3x^3 + 2x$ and $g(x) = \frac{1}{x}$ between $x = 2$ and $x = 5$.

15. (5 points) A pool is draining at a rate of $e^{-0.2t}$ litres per hour. How much water will drain from the pool in the first 30 minutes?

Bonus Question 1. (10 points) Find the volume of the solid obtained by revolving the area bounded by the x and y axes and the function $x = 12(y^2 - y^3)$ around the y -axis.

Bonus Question 2. (5 points) Find $\frac{dy}{dx}$ given that

$$y = \int_{-1000}^x e^{2t} \sin(t^6) dt.$$